

# Gravitinos Tunneling from Black Holes

Alexandre Yale\* and R. B. Mann†

Department of Physics & Astronomy, University of Waterloo  
Waterloo, Ontario N2L 3G1, Canada

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## Abstract

Black hole radiation of gravitinos is investigated as the classically forbidden tunneling of spin-3/2 fermions through an event horizon. We show that all four spin states of the gravitino yield the same emission temperature, and retrieve the Unruh temperature in a Rindler spacetime as well as the Hawking temperature for a Kerr-Newman charged rotating black hole.

## 1 Introduction

In recent years, theorists have begun modeling Hawking radiation as a tunneling effect, offering an alternate conceptual understanding to black hole radiation. Originating with the early work of Kraus and Wilczek [1], the tunneling approach was later refined [2, 3, 4] to show that black hole radiation can indeed be understood in terms of local physics near the horizon. This approach is quite distinct from more global methods such as Wick Rotation methods and Hawking's original method of modeling gravitational collapse [5]. Although the approach was first demonstrated for scalar particles tunneling out of spherically symmetric Schwarzschild black holes, it has been shown to be remarkably versatile, and has been applied successfully to a wide variety of spacetimes, including Kerr and Kerr-Newmann cases [6, 7, 8], black rings [9], the 3-dimensional BTZ black hole [10, 11], Vaidya [12], other dynamical black holes [13], Taub-NUT spacetimes [8], and Gödel spacetimes [14]. One can even recover the Unruh temperature [15] for a Rindler observer [4, 8] using this approach.

All tunneling approaches make use of the WKB approximation to compute the tunneling probability for a classically forbidden trajectory across the horizon. In the present paper, we will consider an infinitesimal path directed in the radial direction from the inside to the outside of the horizon. For the semiclassical action  $I$  of this trajectory to leading order in  $\hbar$

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\*ayale@uwaterloo.ca

†rbmann@sciborg.uwaterloo.ca

(which we set to unity), this yields [1, 16]

$$\Gamma \propto \exp(-2 \text{Im} I) \quad (1)$$

One expects that a black hole should radiate all types of particles, akin to a black body at a well-defined temperature (ignoring grey body effects), such that particles of every spin should appear in the emission spectrum. While the implications of this were studied 30 years ago [17], it has only recently been shown that spin-1/2 fermions can tunnel out of black holes [8], yielding (to leading order in a WKB approximation) the same temperature as one obtains for scalar radiation. As with the scalar case, this approach has also been shown to be quite robust, and has been applied to a broad variety of black hole spacetimes [18, 19, 20].

In this paper we consider radiation of spin-3/2 fermions, or gravitinos. Such particles are predicted to exist in all theories of supergravity [21]. Since local supersymmetry is not a symmetry of the state of our observable universe, it should be broken, and so the gravitino should be massive. As the gravitino obeys the Rarita-Schwinger Equation [22], any one of four possible spin states can be emitted by the black hole. With reference to the radial direction, these components are 3/2, 1/2, -1/2, and -3/2. Since it is not a-priori clear that all components are emitted in the same way, we have a new and interesting check on the tunneling method.

Our approach will be analogous to that taken in the spin-1/2 case [18, 23]. Employing a WKB approximation to the Rarita-Schwinger Equation, we will show that a general structure emerges that ensures all four spin states are radiated by the black hole with at the same Hawking temperature. To keep the discussion more general, we will couple the gravitino to a U(1) gauge field as well as gravity. We apply these results first to Rindler spacetime, confirming that the Unruh temperature is recovered. We then consider the Kerr-Newman case, which had introduced some non-trivial technical features associated with the choice of  $\gamma$  matrices for the spin-1/2 case [23]. We show that the same approach is sufficient in describing gravitino emission to leading order in the WKB approximation<sup>1</sup>.

One of the assumptions of our semi-classical calculation is to neglect any change of angular momentum of the black hole due to the spin of the emitted particle. For zero-angular momentum black holes with mass much larger than the Planck mass this is a good approximation. Furthermore, particles of opposite spin will be emitted in equal numbers statistically, yielding no net change in the angular momentum of the black hole (although second-order statistical fluctuations will be present). We confirm that spin 3/2 fermions are also emitted at the Hawking Temperature. This final result, while not surprising, furnishes an important confirmation of the robustness of the tunneling approach.

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<sup>1</sup>Our approach to gravitino radiation differs from that taken by Jing, who used the Newman-Penrose formalism for a Reissner-Nordström metric [25] to show that the radiating gravitinos necessarily obey a 2nd-order Klein-Gordon-type differential equation in this background

## 2 Gravitinos in a Black Hole Background

The Rarita-Schwinger equation representing the spin-3/2 fermion field will be used in the form [22]

$$i\gamma^\nu (D_\nu + iqA_\nu) \Psi_\mu + m\Psi_\mu = 0 \quad (2a)$$

$$\gamma^\mu \Psi_\mu = 0, \quad (2b)$$

where  $\Psi_\mu \equiv \Psi_{\mu a}$  is a vector-valued spinor of charge  $q$  and mass  $m$ ,  $A_\nu$  represents the electromagnetic potential of the black hole, and the  $\gamma^\mu$  matrices satisfy  $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ . The covariant derivative obeys

$$\begin{aligned} D_\mu &= \partial_\mu + \Omega_\mu \\ \Omega_\mu &= \frac{1}{2} i \Gamma_\mu^{\alpha\beta} \Sigma_{\alpha\beta} \\ \Sigma_{\alpha\beta} &= \frac{1}{4} i [\gamma^\alpha, \gamma^\beta], \end{aligned} \quad (3)$$

where  $\Omega_\mu$  is the spin-connection.

The first equation is the Dirac equation applied to every vector index of  $\Psi$ , while the second is a set of additional constraints to ensure that no ghost state propagates; that is, to ensure that  $\Psi$  represents only spin-3/2 fermions, with no spin-1/2 mixed states.

Working in the context of the path-integral formalism and the WKB approximation, it is known that each path will have a phase of  $\exp(iI/\hbar)$ , where  $I$  is the action corresponding to that path. In the case that we consider here, the infinitesimal radial path across the horizon will dominate, such that we may employ the following ansatz for the wave function:

$$\Psi_\mu = \begin{bmatrix} a_\mu \\ b_\mu \\ c_\mu \\ d_\mu \end{bmatrix} e^{\frac{i}{\hbar} I}, \quad (4)$$

where the  $a_\mu, b_\mu, c_\mu, d_\mu$  are each functions of the spacetime.

Note that the first set of Rarita-Schwinger equations (2a) will yield an equation which can be solved for the action  $I$  independently of the wave function components  $a_\mu, \dots, d_\mu$ . The second set of Rarita-Schwinger equations (2b), on the other hand, will yield four constraints for these wave function components independently of the action, and will have solutions in every spacetime. Hence, as the action is all that we require to find the emission temperature, we need solve only equation (2a). This implies that fermions of every spin will emit at the same temperature, since equation (2b) will never have any effect on the action  $I$ .

Defining the  $\hat{\gamma}$  matrices to be of the chiral form,

$$\hat{\gamma}^0 = \begin{bmatrix} 0 & 1_2 \\ -1_2 & 0 \end{bmatrix} \quad \hat{\gamma}^i = \begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix}, \quad (5)$$

where  $\gamma^\mu = e_I^\mu \hat{\gamma}^I$  are metric-dependent linear combinations of these matrices, we find that the first Rarita-Schwinger equations (2a) can be rewritten

in the form

$$\begin{bmatrix} m & & x_0 + x_3 & x_1 - ix_2 \\ & m & x_1 + ix_2 & x_0 - x_3 \\ -x_0 + x_3 & x_1 - ix_2 & m & \\ x_1 + ix_2 & -x_0 - x_3 & & m \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0, \quad (6)$$

where

$$x_a = e_a^\nu \partial_\nu I. \quad (7)$$

Performing an LU-Decomposition of the matrix above yields a solution independent of the wave function components:

$$\eta^{ab} (e_a^\mu \partial_\mu I) (e_b^\mu \partial_\mu I) - m^2 = 0, \quad (8)$$

where  $\eta = \text{diag}(-1, 1, 1, 1)$  is the Minkowski flat-space metric. This equation is equivalent to the first set of Rarita-Schwinger equations (2a), and will allow us to solve for  $\partial_z I$ , regardless of what  $\Psi$  looks like, and hence independently of (2b). Integrating  $\partial_z I$  along the infinitesimal radial path across the event horizon will yield a complex residue corresponding to the action of the radiation. From this action, the temperature is found using [1, 16]

$$\Gamma \propto \exp(-2\text{Im}I) \propto \exp\left(\frac{-E}{T_H}\right). \quad (9)$$

Note that the above relations hold regardless of the spin-state chosen for gravitino emission. Consequently, all four spin states are radiated with the same Hawking temperature to leading order in WKB. We will now explicitly demonstrate this method in Rindler and Kerr-Newman spacetimes.

### 3 Rindler Spacetime

We first make use of the results of the preceding section in Rindler spacetime, whose simplicity allows one to easily grasp the concept of the method employed. We will here show that it allows us to retrieve the Unruh Temperature [15] for gravitinos. Consider the metric

$$ds^2 = -f(z)dt^2 + dx^2 + dy^2 + \frac{dz^2}{g(z)} \quad (10)$$

where

$$\begin{aligned} f(z) &= \alpha^2 z^2 - 1 \\ g(z) &= \frac{\alpha^2 z^2 - 1}{\alpha^2 z^2} \\ A_\mu &= 0, \end{aligned} \quad (11)$$

which has a singularity at  $z = \alpha^{-1}$ . We will use the  $\gamma^\mu$  matrices in the form

$$\gamma^t = \frac{1}{\sqrt{f(z)}} \hat{\gamma}^0 \quad \gamma^x = \hat{\gamma}^1 \quad \gamma^y = \hat{\gamma}^2 \quad \gamma^z = \sqrt{g(z)} \hat{\gamma}^3, \quad (12)$$

and consider the action to have a solution to (8) of the form

$$I = -Et + W(z) + P(x, y) + K. \quad (13)$$

Inserting (13) into the Rarita-Schwinger equation (2a), we obtain, to leading order in  $\hbar$  such that neither  $\Omega_\mu$  nor the derivatives of the  $a_\mu, \dots, d_\mu$  contribute, the four equations:

$$c_\mu \left( \frac{E}{\sqrt{f}} - W' \sqrt{g} \right) - d_\mu (P_x - iP_y) + a_\mu m = 0 \quad (14a)$$

$$d_\mu \left( \frac{E}{\sqrt{f}} + W' \sqrt{g} \right) - c_\mu (P_x + iP_y) + b_\mu m = 0 \quad (14b)$$

$$a_\mu \left( \frac{E}{\sqrt{f}} + W' \sqrt{g} \right) + b_\mu (P_x - iP_y) - c_\mu m = 0 \quad (14c)$$

$$b_\mu \left( \frac{E}{\sqrt{f}} - W' \sqrt{g} \right) + a_\mu (P_x + iP_y) - d_\mu m = 0. \quad (14d)$$

The constraints (2b) will also give us the relations

$$\frac{c_1}{\sqrt{f}} + d_2 - id_3 + \sqrt{g}c_4 = 0 \quad (15a)$$

$$\frac{d_1}{\sqrt{f}} + c_2 + ic_3 - \sqrt{g}d_4 = 0 \quad (15b)$$

$$\frac{-a_1}{\sqrt{f}} + b_2 - ib_3 + \sqrt{g}a_4 = 0 \quad (15c)$$

$$\frac{-b_1}{\sqrt{f}} + a_2 + ia_3 - \sqrt{g}b_4 = 0 \quad (15d)$$

between the various components of the wave function. These are not important here as our equations will yield a solution for the action that is independent of the wave function, meaning that these new constraints cannot have any effect on the action provided they allow a non-zero solution  $\Psi$ , which they always will.

Solving equations (14) exactly and generally, we get the same solution as (8):

$$\left( \frac{E}{\sqrt{f}} - W' \sqrt{g} \right) \left( \frac{E}{\sqrt{f}} + W' \sqrt{g} \right) - (P_x - iP_y)(P_x + iP_y) + m^2 = 0. \quad (16)$$

As we approach the horizon, we find that  $f(z)$  and  $g(z)$  both go to zero, while the other terms (besides  $W'$ ) are constant, and therefore get

$$W'_\pm = \frac{\pm E}{\sqrt{fg}}. \quad (17)$$

This solution will be valid for any non-zero  $\Psi$  satisfying (15), whose solution space generally is 12-dimensional.

We then integrate (17) over our path through the event horizon, which contains a pole that will be the sole imaginary contribution to the action. This yields:

$$ImW_\pm(z) = \frac{\pm \pi E}{\sqrt{f_z(z_0)g_z(z_0)}} = \frac{\pi E}{2\alpha}. \quad (18)$$

As  $W'$  corresponds to the momentum of the particle, one finds that  $W_+$  corresponds to an outgoing particle, whereas  $W_-$  corresponds to an incoming one. By forcing to unity the probability that the incoming particle is absorbed and using the fact that  $W_+ = -W_-$ , we find

$$\begin{aligned}\Gamma &\propto \exp(-2ImI) = \frac{\exp(-2(ImW_+ + ImP(x, y) + ImK))}{\exp(-2(ImW_- + ImP(x, y) + ImK))} \\ &= \exp(-4ImW_+) \\ &= \exp\left(\frac{-2\pi}{\alpha}E\right),\end{aligned}\tag{19}$$

which gives us the expected Unruh temperature [15]:

$$T_H = \frac{\alpha}{2\pi}.\tag{20}$$

## 4 Kerr-Newman Black Hole Spacetime

We next consider applying this method to a general Kerr-Newman black hole spacetime. The metric is given by

$$ds^2 = -f dt^2 + \frac{dr^2}{g} - 2H dt d\phi + K d\phi^2 + \Sigma d\theta^2,\tag{21}$$

where

$$\begin{aligned}A_\mu &= \frac{-er}{\Sigma(r)} ((dt)_\mu - \alpha \sin^2(\theta)(d\phi)_\mu) \\ f(r, \theta) &= \frac{\Delta(r) - \alpha^2 \sin^2(\theta)}{\Sigma(r, \theta)} \\ g(r, \theta) &= \frac{\Delta(r)}{\Sigma(r, \theta)} \\ H(r, \theta) &= \frac{\alpha \sin^2(\theta)(r^2 + \alpha^2 - \Delta(r))}{\Sigma(r, \theta)} \\ K(r, \theta) &= \frac{(r^2 + \alpha^2)^2 - \Delta(r)\alpha^2 \sin^2(\theta)}{\Sigma(r, \theta)} \sin^2(\theta) \\ \Sigma(r, \theta) &= r^2 + \alpha^2 \cos^2(\theta) \\ \Delta(r) &= r^2 + \alpha^2 + e^2 - 2Mr = (r - r_-)(r - r_+).\end{aligned}\tag{22}$$

We assume a non-extremal black hole,  $M^2 > \alpha^2 + e^2$ , such that we have two horizons at

$$r_\pm = M \pm \sqrt{M^2 - \alpha^2 - e^2}.\tag{23}$$

To simplify the notation, we will also use the functions

$$F(r, \theta) = f(r, \theta) + \frac{H^2(r, \theta)}{K(r, \theta)} = \frac{\Delta(r)\Sigma(r, \theta)}{(r^2 + \alpha^2)^2 - \Delta(r)\alpha^2 \sin^2(\theta)}\tag{24a}$$

$$\Omega_H = \frac{H(r_+, \theta)}{K(r_+, \theta)} = \frac{a}{r_+^2 + \alpha^2},\tag{24b}$$

where  $\Omega_H$  corresponds to the angular velocity of the black hole. We will use the following  $\gamma^\mu$  matrices [23] :

$$\begin{aligned}\gamma^t &= \frac{1}{\sqrt{F(r, \theta)}} \hat{\gamma}^0 & \gamma^r &= \sqrt{g(r, \theta)} \hat{\gamma}^3 & \gamma^\theta &= \frac{1}{\sqrt{\Sigma(r, \theta)}} \hat{\gamma}^1 \\ \gamma^\phi &= \frac{1}{\sqrt{K(r, \theta)}} \left( \hat{\gamma}^2 + \frac{H(r, \theta)}{\sqrt{F(r, \theta)K(r, \theta)}} \hat{\gamma}^0 \right),\end{aligned}\quad (25)$$

where the  $\hat{\gamma}^i$  are the chiral Minkowski matrices (5). The action will in this case take the form

$$I = -Et + W(r, \theta) + J\phi. \quad (26)$$

If we then input our wave function (4) into the Rarita-Schwinger equations (2a), we get, again to leading order in  $\hbar$ , such that neither  $\Omega_\mu$  nor the derivatives of the  $a_\mu, \dots, d_\mu$  contribute, equations similar to those found for the Rindler spacetime:

$$c(\zeta - W_r \sqrt{g}) + d \left( \xi - \frac{W_\theta}{\sqrt{\Sigma}} \right) + am = 0 \quad (27a)$$

$$d(\zeta + W_r \sqrt{g}) + c \left( -\xi - \frac{W_\theta}{\sqrt{\Sigma}} \right) + bm = 0 \quad (27b)$$

$$a(\zeta + W_r \sqrt{g}) - b \left( -\xi - \frac{W_\theta}{\sqrt{\Sigma}} \right) - cm = 0 \quad (27c)$$

$$b(\zeta - W_r \sqrt{g}) - a \left( \xi - \frac{W_\theta}{\sqrt{\Sigma}} \right) - dm = 0, \quad (27d)$$

where

$$\zeta \equiv \frac{E}{\sqrt{F}} + \frac{qer}{\Sigma \sqrt{F}} - \left( J + \frac{qer}{\Sigma} a \sin^2(\theta) \right) \frac{H}{K \sqrt{F}} \quad (28a)$$

$$\xi \equiv i \left( \frac{J}{\sqrt{K}} + \frac{qer}{\Sigma \sqrt{K}} a \sin^2(\theta) \right). \quad (28b)$$

Once again, the second set of Rarita-Schwinger equations (2b) will give us additional relations between the various vector components of the wave function, which are not important here as our solution for the action will be independent of such relations:

$$\frac{c_1}{\sqrt{F}} + \sqrt{g}c_2 + \frac{d_3}{\sqrt{\Sigma}} + \frac{Hc_4}{K\sqrt{F}} - \frac{id_4}{\sqrt{K}} = 0 \quad (29a)$$

$$\frac{d_1}{\sqrt{F}} - \sqrt{g}d_2 + \frac{c_3}{\sqrt{\Sigma}} + \frac{Hd_4}{K\sqrt{F}} + \frac{ic_4}{\sqrt{K}} = 0 \quad (29b)$$

$$\frac{-a_1}{\sqrt{F}} + \sqrt{g}a_2 + \frac{b_3}{\sqrt{\Sigma}} - \frac{Ha_4}{K\sqrt{F}} - \frac{ib_4}{\sqrt{K}} = 0 \quad (29c)$$

$$\frac{-b_1}{\sqrt{F}} - \sqrt{g}b_2 + \frac{a_3}{\sqrt{\Sigma}} - \frac{Hb_4}{K\sqrt{F}} + \frac{ia_4}{\sqrt{K}} = 0. \quad (29d)$$

Equations (27) may be rewritten in a way similar to (8), such that they become independent of the wave function components  $a_\mu, \dots, d_\mu$  :

$$(\zeta - W_r \sqrt{g})(\zeta + W_r \sqrt{g}) - \left( \frac{W_\theta}{\sqrt{\Sigma}} - \xi \right) \left( \frac{W_\theta}{\sqrt{\Sigma}} + \xi \right) + m^2 = 0. \quad (30)$$

Since the action may be reduced to an infinitesimal path from inside to outside the horizon,  $\theta$  may only take values between  $\theta_0 - \varepsilon$  and  $\theta_0 + \varepsilon$ . Hence, we make the approximation  $\theta = \theta_0$ , thus assuming that  $\theta$  is a constant of motion, which in turn forces  $W_\theta$  to be constant. Then, expanding (30) near the horizon  $r \rightarrow r_+$  and solving for  $W_r$ , we see that only  $\zeta$  contributes, as it rapidly increases to infinity near the horizon while other terms do not. This allows us to find a solution similar to that for emission of Dirac particles [18]:

$$W_{r\pm} = \frac{\pm 1}{\sqrt{g}} \zeta = \frac{\pm \left( E - J\Omega_H + \frac{qer_+}{r_+^2 + \alpha^2} \right)}{\sqrt{F_r(r_+)g_r(r_+)(r - r_+)}}. \quad (31)$$

Integrating (31) yields

$$W_\pm = \frac{\pm \pi i \left( E - J\Omega_H + \frac{qer_+}{r_+^2 + \alpha^2} \right) (r_+^2 + \alpha^2)}{2r_+ - 2M}. \quad (32)$$

which is independent of the particular choice of  $\theta_0$ . Thus, taking again  $\Gamma \propto \exp(-4ImW_+)$ , we find

$$\Gamma \propto \exp \left( -2\pi \frac{r_+^2 + \alpha^2}{r_+ - M} \left( E - J\Omega_H + \frac{qer_+}{r_+^2 + \alpha^2} \right) \right), \quad (33)$$

giving us the expected Hawking temperature for a charged rotating black hole,

$$T_H = \frac{1}{2\pi} \frac{r_+ - M}{r_+^2 + \alpha^2} = \frac{1}{2\pi} \frac{\sqrt{M^2 - \alpha^2 - e^2}}{2M (M + \sqrt{M^2 - \alpha^2 - e^2}) - e^2}. \quad (34)$$

## 5 Conclusions

We have shown that gravitinos, as described by the Rarita-Schwinger equation in a curved spacetime, can tunnel out of event horizons. The tunnelling formalism applies equally well to this situation, and we found that gravitinos attain the familiar Unruh temperature in a Rindler spacetime. We likewise recover the Hawking temperature for gravitino radiation from a Kerr-Newman black hole. All four spin components yield the same temperature to leading order in a WKB approximation. These results are in agreement with those obtained by applying this method to spin-1/2 fermions [18, 23], as well as with results obtained by using the Newman-Penrose approach on gravitinos [25] radiating from a Reissner-Nordström black hole.

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